A SIGNAL PROCESSING SCHEME FOR REDUCING THE CAVITY PULLING FACTOR IN PASSIVE HYDROGEN MASERS

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ABSTRACT

A passive hydrogen maser operates so as to cause a signal frequency, fs, to satisfy a selected criterion. The frequency fs which satisfies the criterion depends on the cavity resonance frequency, fc. The derivative of fs with respect to fc for fc = fs is called the PULLING FACTOR. Theoretically this factor can be zero with a computational criterion making use of complex signal voltage samples taken at several frequencies.

INTRODUCTION

Consider the oscillator control servo for a passive hydrogen maser. A signal frequency fs is synthesized from the oscillator output frequency. The oscillator frequency is controlled so as to cause fs to equal Fo, the (perturbed) hydrogen transition frequency. Possible servo control criteria include:

a. The difference in maser transfer magnitudes at frequencies equally spaced above and below fs equals zero, the frequency spacing being less than the hydrogen linewidth.

b. The maser transfer phase at frequency fs equals zero.

c. The difference between the maser transfer phase at fs and the mean of the phases at frequencies equally spaced above and below fs equals zero, the frequency spacing being very much larger than the hydrogen linewidth.

The signal frequency fs which satisfies the selected criterion depends on the value of the cavity resonant frequency fc. The derivative of fs with respect to fc, under servo control, is the PULLING FACTOR.

Pulling factors for criteria (a) and (b) above are derived in Section 11 of reference 1, and are substantially different for the two cases. For criterion (c) it can be shown that the pulling factor is closely the ratio of the cavity and hydrogen line Q's, also differing from criteria (a) and (b). It appears then that, at least in part, the pulling factor is a result of the method used to cause fs to approximate the hydrogen transition frequency.

The pulling factors mentioned above were all derived using a model of the steady state complex microwave field as a function of frequency given by Lesage, Audoin and Tetu in reference 1 (1979). Using the same model it is possible to devise a criterion for which the derivative of fs with respect to fc is zero when fc = fs. Unlike criteria (a),(b) and (c) above it makes use of both magnitude and phase of transfer measurements at fs and at frequencies equally spaced above and below fs.

PROPOSED DEMONSTRATION HARDWARE

At the present time the proposed frequency error criterion has not been demonstrated experimentally. An experiment should evaluate at least the following:

Cavity pulling characteristic
Frequency error noise due to receiver noise
Bias due to error estimation algorithm.

Figure 1 is a simplified block diagram for demonstration of the signal processing scheme. Some functions such as digital/analog conversion and cavity tuning are not shown where needed. The process, under computer control, includes the following steps:

switch the maser input signal (V1) to frequencies fa, fs, and fb in sequence (FREQUENCY SYNTHESIZER)

measure complex voltage ratios at these frequencies (NETWORK ANALYZER)

acquire and filter complex samples (COMPUTER)

perform frequency error computations (COMPUTER)

perform servo loop filter functions (COMPUTER)

correct oscillator frequency error (COMPUTER).

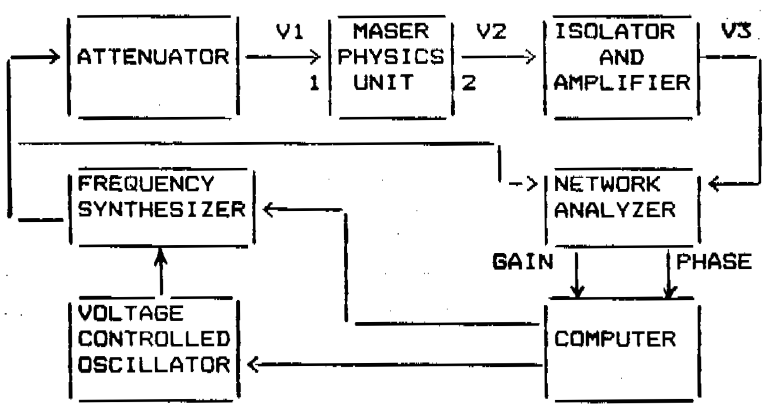


FIGURE 1 CONCEPTUAL BLOCK DIAGRAM

CAVITY RESONATOR INVERSE TRANSFER FUNCTION

The steady-state complex transfer function of the passive maser is the ratio of the voltages V2 and V1 defined in Figure 2. For a high-Q single-pole cavity resonator without atomic hydrogen the function consists of a fraction with a constant numerator. The denominator is unity plus an imaginary term vc which is a linear function of frequency. For practical reasons Figure 2 includes an isolator and an amplifier whose output is V3. We consider measurements of complex values of the ratio of V3 and V1.

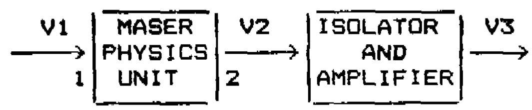
In equation (1) the symbol Hc is introduced, which is the ideal cavity transfer function denominator, Equation (2). In the definition of vc (3) we see that the imaginary term is equal to the twice the difference between the signal frequency and the cavity resonant frequency divided by the cavity bandwidth.

The complex plot of Figure 2 shows the contour of Hc as frequency is varied, with a particular value indicated by *. The simplicity and linearity of Hc suggests the use of inverse transfer functions for parameter estimation from measurement data.

If we obtain an inverse transfer function from measured voltages as V1 / V3 it will consist of Hc multiplied by a complex number Ho which results from cavity insertion loss, various phase shifts, and gains and losses associated with the paths from the measurement junctions to the maser . In the following development we assume that variation of Ho over the band of frequencies of interest is negligible.

V1 INCIDENT VOLTAGE TO PORT 1

V2 INCIDENT VOLTAGE FROM PORT 2



FOR THE IDEAL CAVITY WITHOUT ATOMIC HYDROGEN:

(1) V3 / V1 = 1/ Ho Hc

Ho COMPLEX CONSTANT TO ACCOUNT FOR CAVITY INSERTION LOSS, AMPLIFIER GAIN, VARIOUS PHASE LAGS

(2) Hc = 1 + j vc

(3) vc = 2 (f − fc) / Bc

f FREQUENCY, HERTZ

fc CAVITY RESONANT FREQUENCY

Bc CAVITY HALF-POWER BANDWIDTH

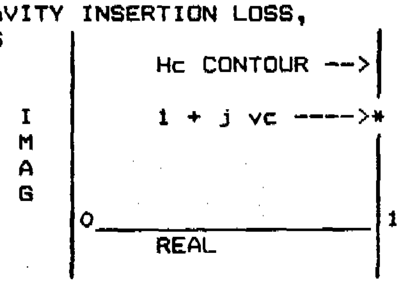


FIGURE 2 CAVITY RESONATOR INVERSE TRANSFER FUNCTION

Although Ho is unknown in the practical measurement situation, it is possible to find (estimate) its complex value and other parameters from measurements of V1/V3 at two frequencies whose difference is known. Let the two frequencies fa and fb be equally spaced above and below a frequency fs as illustrated in Figure 3. We will call the measured inverse transfer at these frequencies Ha and Hb, respectively. The corresponding values of Hc will be Hca and Hcb, as in the table.

In equation (4) we take the mean of Ha and Hb, which is Ho times the mean of Hca and Hcb, and find the product Ho Hcs where Hcs is the value of Hc at frequency fs, to be used later. From the difference between Ha and Hb we can find Ho multiplied by an imaginary constant, as in (5). We will use this expression later to remove the angle rotation due to Ho from calculations.

Dividing (4) by (5) produces an estimate of Hcs/(j 2 F1 / Bc). The real part of this quotient (6) is the difference between fs and the cavity resonance frequency fc, divided by F1 (known). The imaginary part (7) is the cavity bandwidth divided by 2 F1. Multiplying (5) and (7) and + j gives the complex value for Ho, as in (8). Hence two complex ratios suffice to characterize the ideal cavity resonator and the measured path:

Ho	the inverse complex gain of the path including the cavity insertion loss
Bc	the cavity bandwidth (fc divided by loaded Q)
fs - fc	the cavity tuning error in Hertz, if fs is the desired resonant frequency.

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fs    SIGNAL FREQUENCY, VARIABLE       |<-- F1-->|<-- F1-->|
                                   ____|_________|_________|______
F1    SPACING FREQUENCY, CONSTANT      fb        fs        fa      f
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FREQUENCY	V1 / V3	Hc
fa = fs + F1	Ha = Ho Hca	Hca = 1 + j 2 (fs + F1 - fc) / Bc
fb = fs - F1	Hb = Ho Hcb	Hcb = 1 + j 2 (fs - F1 - fc) / Bc

(4) $(Ha + Hb)/2 = Ho\ (1 + j\ 2\ (fs - fc)\ /\ Bc\ ) \quad = \quad Ho\ Hcs$

(5) $(Ha - Hb)/2 = \quad Ho\ (j\ 2\ F1\ /\ Bc)$

(6) $\mathrm{REAL}\ (\ (Ha + Hb)/(Ha - Hb)\ ) = \quad (fs - fc)\ /\ F1$

(7) $\mathrm{IMAG}\ (\ (Ha + Hb)/(Ha - Hb)\ ) = \quad -\ Bc\ /\ 2F1$

(8) $(\ -\ j\ Bc\ /\ 2\ F1)\ (Ha - Hb)\ /\ 2 = \quad Ho$

FIGURE 3 CAVITY SIDE-FREQUENCY TRANSFER RELATIONSHIPS

MASER INVERSE TRANSFER FUNCTION

The passive maser transfer function with atomic hydrogen present can be readily derived from steady-state microwave field equations (35) through (38) from the paper of Lesage, Audoin, and Tetu in the Proceedings of the 33rd Annual Symposium on Frequency Control, 1979, pages 515 through 535 (reference 1). They assume that the cavity mistuning is small, and that the difference between the microwave frequency and the cavity resonant frequency is a small fraction of the cavity bandwidth.

Equation (9) introduces the symbol Hm for the denominator of the maser transfer function, multiplied by Ho as before. Equation (10) gives the function Hm, consistent with the field equations of reference 1, although different in appearance. The symbols α, S, and T2 are used as defined in reference 1.

The denominator of the maser transfer function is the sum of the terms presented above for the cavity transfer and a complex term due to the hydrogen atoms. The hydrogen contribution to the inverse transfer function is proportional to the parameter α. When equal to zero there is no hydrogen contribution. When greater than unity the maser will oscillate. Saturation, represented by the factor S, increases with microwave field amplitude and decreases with absolute signal frequency difference from the hydrogen transition frequency, Fo.

The complex portion of the hydrogen contribution to Hm has a ratio of imaginary to real parts which is proportional to the frequency difference f - Fo. If f is the frequency of a signal which is intended to equal Fo then the value of this ratio is proportional to the frequency error. The proportionality factor can be calculated from prior knowledge of the transverse relaxation time, T2. For control purposes it need not be known precisely.

FOR THE PASSIVE MASER WITH ATOMIC HYDROGEN:

(9) V1 / V3 = Ho Hm WHERE:

(10) Hm = 1 + j vc − (α /(1 + S))/ (1 + j 2 π T2 (f − Fo))

DERIVED FROM (35), (36) AND (37) OF REFERENCE 1, WHERE

α PARAMETER WHICH CHARACTERIZES OPERATING CONDITIONS RELATIVE TO THRESHOLD OF OSCILLATION. (22), REF. 1

S SATURATION FACTOR OF THE ATOMIC TRANSITION (38), REF. 1

T2 TRANSVERSE RELAXATION TIME OF HYDROGEN ATOMS

Fo ATOMIC TRANSITION FREQUENCY

FIGURE 4 MASER INVERSE TRANSFER FUNCTION

Figure 5 introduces the symbol Hh for the hydrogen contribution to the inverse maser transfer Hm, and in (12) expresses it in terms of variables ah and vh. The angle of Hh is a function only of vh (14) which in turn is proportional to f - Fo.

Let fs be the frequency that is to be controlled to equal Fo, and let vhs be the value of vh at fs. Let Hms be the value of Hm and Hhs the value of Hh at frequency fs.

Earlier Hca and Hcb were defined for the cavity without atomic hydrogen. Now we will assume that fa and fb are sufficiently far from Fo that the effect of the hydrogen atoms on Hm at these frequencies is negligible. We showed in Figure 3 that the mean of Hca and Hcb equals Hcs, which cannot be measured directly in the presence of the atomic hydrogen.

If the computed value of Hcs is subtracted from the maser inverse transfer Hms the result is Hhs (15). The imaginary part of Hhs divided by its real part gives vh (16) which in turn is proportional to the frequency error fs - Fo, independent of the value of Hcs which depends on the cavity tuning error fc - fs.

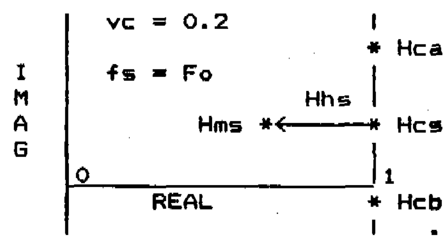
The complex plot of Figure 5 shows three points Hms, Hca, and Hcb representing measured values, and Hcs representing a computed value. The location of Hcs indicates a cavity tuning error -0.1 times the cavity bandwidth (2(fs - fc)/Bc = 0.2). The line from Hcs to Hms is horizontal, hence zero imaginary part of Hhs, indicating that fs = Fo in this exammple.

(11) $Hm = Hc + Hh$, WHERE:

(12) $Hh = -\, ah \,/\, (1 + j\, vh)$

(13) $ah = \alpha \,/\, (1 + S)$

(14) $vh = 2\pi\ T2\ (f - Fo)$



AT FREQUENCY fs, Hm = Hms and Hh = Hhs BY DEFINITION

(15) $Hhs = Hms - Hcs = Hms - (Hca - Hcb) / 2$

(16) $-\ IMAG\ (Hhs) \,/\, REAL\ (Hhs) = vh = 2\pi\ T2\ (fs - Fo)$

FIGURE 5 HYDROGEN CONTRIBUTION TO MASER INVERSE TRANSFER

FREQUENCY ERROR COMPUTATION

The frequency error computation would consist of equations (15) and (16) were it not for the angle of Ho, due to the phase components of the paths which connect the measurement junctions with the maser. For actual measurements the whole plot of Figure 5 would be rotated counterclockwise through the angle of Ho.

Figure 6 shows relationships which permit computation of the ratio of the real and imaginary components of Hhs from measurements. In the table of Figure 6 we name the measured inverse transfer ratios Hs, Ha, and Hb, each containing the factor Ho. The expressions for Hms, Hca, and Hcb are also included.

H1 defined in (17) corresponds to (15), but with the factor Ho included. H2 in (18) is the complex value from (5) of Figure 3 with the sign of its imaginary part reversed. Multiplying H1 and H2 (19) then produces Hhs multiplied by two real numbers and rotated through $-\pi/2$. The real factors are 4 F1/ Bc and (Ho CONJUGATE (Ho)).

The value of vhs in terms of the components of Hhs is reproduced in (20), and the equivalent relationship in terms of components of H3 is given in (21). The frequency error computation in the presence of Ho consists of (17), (18), (19), and (21).

FREQUENCY	V1 / V3	H
fs	Hs = Ho Hms	Hms = Hcs - ah / (1 + j vhs)
fa = fs + F1	Ha = Ho Hca	Hca = 1 + j 2 (fs + F1 - fc) / Bc
fb = fs - F1	Hb = Ho Hcb	Hcb = 1 + j 2 (fs - F1 - fc) / Bc

(17) $H1 = Hs - (Ha + Hb)/2 = Ho\ Hms - Ho\ Hcs = Ho\ Hhs$

(18) $H2 = \mathrm{CONJUGATE}\ (Ha - Hb) = \mathrm{CONJUGATE}\ (\ j\ (4\ F1\ /\ Bc)\ Ho)$

(19) $H3 = H1\ H2 = -\ j\ Hhs\ (4\ F1/Bc)\ (\ Ho\ \mathrm{CONJUGATE}\ (Ho)\ )$

(20) $vhs = -\ \mathrm{IMAG}\ (Hhs)\ /\ \mathrm{REAL}\ (Hhs)$

(21) $2\pi\ T2\ (fs - Fo) = \mathrm{REAL}\ (H3)\ /\ \mathrm{IMAG}\ (H3)$

FIGURE 6 SOLUTION FOR SIGNAL FREQUENCY ERROR

DISCUSSION

The signal processing scheme described here for reducing the cavity pulling factor of a passive maser appears to offer the following:

1. Reduction of errors due to uncompensated cavity resonance varitions (temperature, etc.)

2. Reduction of errors due to receiver noise and electronic system imperfections in the cavity servo

3. The possibility of operating with a temperature-stable cavity without autotuning.

The method may also be of use in monitoring the cavity drift in large active masers without autotuning.

The method has apparent limitations. The frequency-error computation is based on six measured real values (three complex values) compared to three in the case of criterion (c), and two in criterion (a). Each of these values includes a contribution due to receiver noise. Noise analysis for an oscillator control servo using this method has not yet been accomplished. Some rough reasoning indicates that the frequency-error noise density will be greater than for criterion (c).

The model assumes a single-mode cavity resonator with ideal transfer function symmetry. Sensitivity to unwanted cavity modes, non-ideal microwave circuits, and filters in the common signal path is not known. Additional circuit transfer function elements may be accomodated by taking measurements at additional frequencies, but with the penalty of additional noise.

The hydrogen influence at the side frequencies fa and fb was neglected in the derivations. The real part of this influence is less than the square of 1/vh evaluated at the side frequency. The absolute value of the imaginary part is less than 1/vh, closely equal and opposite at the two frequencies. In the absence of saturation these would cause no errors. At actual operating levels they will cause higher order pulling, showing up for sufficiently large cavity tuning error, vcs. The error vcs = 0.2 in Figure 5 is for illustration but is undoubtedly far too large for satisfactory accuracy of the assumption.

While it is not clear whether this error-detection scheme is advantageous, the derivations imply that cavity pulling is not an unavoidable perturbation of the atomic hydrogen emission but is a result of the scheme used to approximate its frequency.

ACKNOWLEDGEMENTS

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REFERENCES

1. P. Lesage, C. Audoin, and M. Tetu, "Amplitude Noise in Passively and Actively Operated Masers," Proceedings 33rd Annual Symposium on Frequency Control, 1979, pp. 515 - 535.

2. F. L. Walls, "Frequency Standards Based on Atomic Hydrogen," Proceedings of the IEEE, vol. 74, no. 1, January 1986, pp. 142-146.